

NONLINEAR IDENTIFICATION OF CASCADED TWO TANK SYSTEM

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Abstract: Design of controller for the process operation is typically based on the availability of the best model. Development of empirical model is incorporated with many assumptions and approximations. System identification is developed to alleviate this problem by considering the input and output data of the process for the model development. The cascaded tank process is highly nonlinear and non-minimum phase process and often its working conditions are variable. Due to inadequacy of linear system identification to capture the dynamic of process, nonlinear system identification is incorporated. Identification of Cascaded two tank process using Wiener model is reported. Wiener model parameters are estimated using recursive prediction error method.

Keywords: *Nonlinear models, Cascaded two tank process, Wiener Model, Recursive Prediction Error Method.*

I. INTRODUCTION

System identification is the process of constructing mathematical model from measured data. Most of the physical systems are nonlinear in nature. Linear Identification is not suitable for nonlinear systems. Identification of such nonlinear system is an important research area. Numerous methods are available for the identification of nonlinear systems includes [1], [2], [3], [4]. Chen et al describes the identification of discrete-time nonlinear systems using neural networks [1], Bohlin reported the grey-box identification to design a nonlinear model for the input/output data from strip steel rinsing process [2], Genetic Programming are applied to the identification of the nonlinear structure of a dynamic model from experimental data are proposed by Gray et al [3], and Li et al presented Fast Recursive Algorithm for the identification of nonlinear dynamic systems using linear in the parametric models with significant reduction in computational complexity and is numerically more stable than Orthogonal Least Square [4]. The Identification of nonlinear model can be represented by Wiener model using Volterra series expansion characterization is given by Billings et al [5]. This approach overcomes many of the disadvantages associated with black-box identification and provides a very concise description of the process. Wigren proposed a recursive prediction error method has been derived from a SISO Wiener model [6]. A drawback is that the suggested scheme cannot be applied in a straightforward manner when the system is not asymptotically stable.

To overcome this drawback, use Gaussian rather than bounded input in order to appropriately estimate the linear subsystem based on the observation data is given by Hu et al in [7] and [8]. All estimates for coefficients of the linear subsystem as well as for the values of the nonlinear block are given recursively with the help of Stochastic Approximation algorithms. The difference between these papers is the effective use of available information. In [7], only a part of data is used for the identification purpose while in [8] the whole sequence of data is employed. In this paper, the cascaded two tank process is identified using Recursive Prediction Error algorithm for Wiener model.

II. SYSTEM DESCRIPTION

The Cascaded two tank process is taken as a benchmark for the nonlinear identification is shown in Fig.1. Water is pumped to the upper tank. Control signal is applied to the pump in terms of voltage. The resulting water level is measured in the lower tank. Open outlet in the lower tank is demonstrated the nonlinear dynamics in the process due to the changes in the level of the tank. Voltage applied (u) to the pump and lower tank level (y_m) is considered as input and output data respectively which is available in [9].

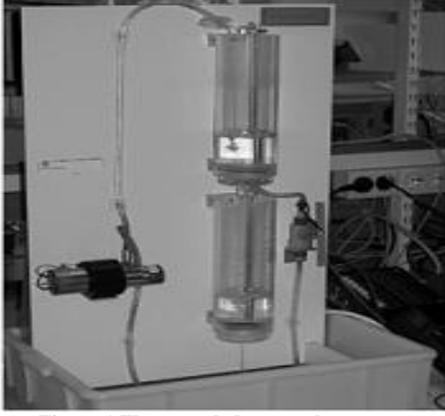


Figure 1 The cascaded two tank process

III. NONLINEAR MODELS

System Identification represents the mathematical relationships between the system's inputs $u(t)$ and outputs $y(t)$. It can be used to compute the current output from previous inputs and outputs. The general form of model in discrete time is:

$$y(t) = f(u(t-1), y(t-1), u(t-2), y(t-2), \dots) \quad (1)$$

Such a model is nonlinear if the function f is a nonlinear function. Block-oriented models are used to model nonlinear systems that can be represented by the interconnections of linear dynamics and static nonlinear elements. Wiener model consists of a linear dynamic system followed by a static nonlinearity, is shown in Fig.2.

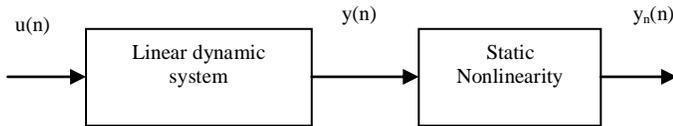


Figure 2 Wiener Model Structure

Wiener models arise in practice whenever a measurement device has a nonlinear characteristic. If the output of a system depends nonlinearly on its inputs, it might be possible to decompose the input-output relationship into two or more interconnected elements.

IV. RECURSIVE PREDICTION ERROR IDENTIFICATION OF WIENER MODEL

The Recursive identification of wiener model is obtained by Discontinuous Piecewise Linear Function [10], using optimal local linear models [11], two segment polynomial nonlinearities [12], non invertible piecewise linear function by following the use of ordinary recursive least square method [13]. A parameterization of a SISO Wiener model is

presented with an algorithm for simultaneous recursive identification. Due to the cascade structure of the Wiener model, it is generally not possible to identify the linear dynamics independent of the static nonlinearity [14].

The parameter vector θ is partitioned as $\theta = (\theta_l^T \theta_n^T)^T$, where θ_l is the parameter of linear block and θ_n is the parameter of static nonlinearity. The SISO linear dynamic block of the model is described by

$$\hat{y}(t|\theta_l) = \frac{B(q^{-1})}{F(q^{-1})} u(t) \quad (2)$$

Where $u(t)$ is the input signal and $\hat{y}(t|\theta_l)$ is the output.

$$B(q^{-1}) = b_1 q^{-1} + \dots + b_{nb} q^{-nb} \quad (3)$$

$$F(q^{-1}) = f_1 q^{-1} + \dots + f_{nf} q^{-nf} \quad (4)$$

$$\theta_l = (f_1 \dots f_{nf} b_1 \dots b_{nb})^T \quad (5)$$

$$\varphi(t, \theta_l) = ((-\hat{y}(t-1|\theta_l) \dots - \hat{y}(t-n_f|\theta_l) u(t-1) \dots u(t-nb))^T \quad (6)$$

The output from the nonlinear block is modeled as

$$\hat{y}_n(t|\theta_l, \theta_n) = f_n(\theta_n, \hat{y}(t|\theta_l)) \quad (7)$$

Where $f_n(\dots)$ is a known function of θ_n and of $\hat{y}(t|\theta_l)$. In this paper, a piecewise linear model of the static nonlinearity will be used. Therefore $f_n(\theta_n, \hat{y}(t|\theta_l))$ is modeled as,

$$f_n(\theta_n, \hat{y}(t|\theta_l)) = k_0 \hat{y}(t|\theta_l) + f_{n,0} \hat{y}(t|\theta_l) \in I_0 \quad (8)$$

In that interval, where $f_{n,0}$ is the bias parameter in I_0 . In order to describe the model outside I_0 , the following set user chosen grid points is introduced,

$$\text{grid} = (y_{-k} \ y_{-k+1} \ \dots \ y_0 \ y_0 \ \dots \ y_{k-1} \ y_k) \quad (9)$$

The complete parameter vector θ_n is therefore given by

$$\theta_n = (f_{n,0} \ f_{n,-k} \ \dots \ f_{n,-1} \ f_{n,1} \ \dots \ f_{n,k})^T \quad (10)$$

The complete RPEM for simultaneous recursive identification of the linear dynamics and the static nonlinearity is

$$\varepsilon_n(t) = y_n(t) - \hat{y}_n(t) \quad (11)$$

Torbjorn Wigren developed the MATLAB code for Recursive prediction error method which is considered for our study [6],[15].

Model Validation:

The output of the model produces the best fit is computed using the following equation (12). In this equation, y_m is the measured output, y_{mmod} is the simulated or predicted model output with 100% corresponds to a perfect fit.

$$\text{Best fit} = 1 - \left(\frac{\|y_m - y_{mmod}\|}{\|y_m - \text{mean}(y_m)\|} \right) * 100 \quad (12)$$

V. RESULTS AND DISCUSSION

The cascaded tank process input u and output y_m data of 2500 values is plotted with respect to time is shown in Fig.3.

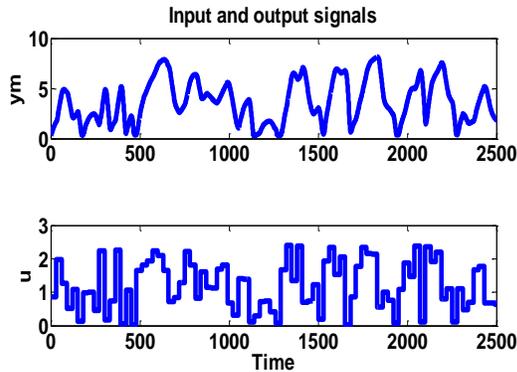


Figure 3 Input Output response of Cascaded two tank process

The evolution of the parameter estimates of linear and nonlinear parts are shown in Fig.4 and Fig.5

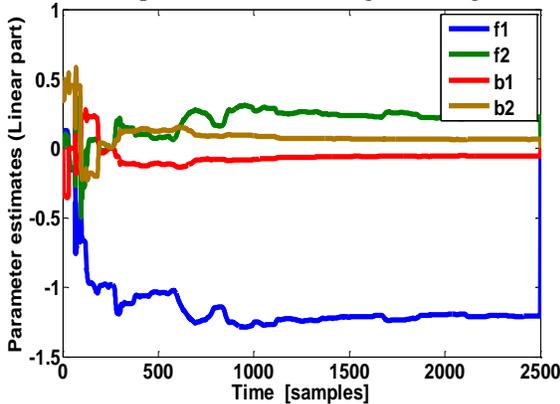


Figure 4 Parameter Estimates of Linear part

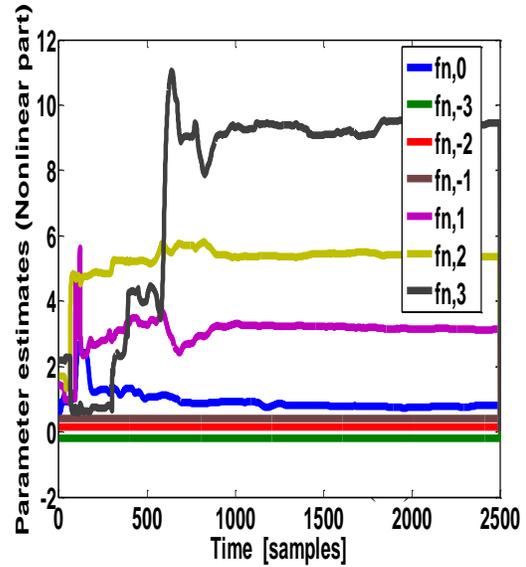


Figure 5 Parameter Estimate of Nonlinear part

The output signal of the system (y_m) and the simulated model (y_{mmod}) using running parameter estimates as a function of time is shown in Fig.6. The simulated output errors are similar to the output with prediction error. The difference is that the simulated output errors are generated with the fixed parameter vector, obtained at the end of the identification run. This RPEM algorithm validates the estimated Wiener model by plotting the output signal from the system and the estimated model using the final value of the parameter vector with the residual signal is plotted in Fig.7

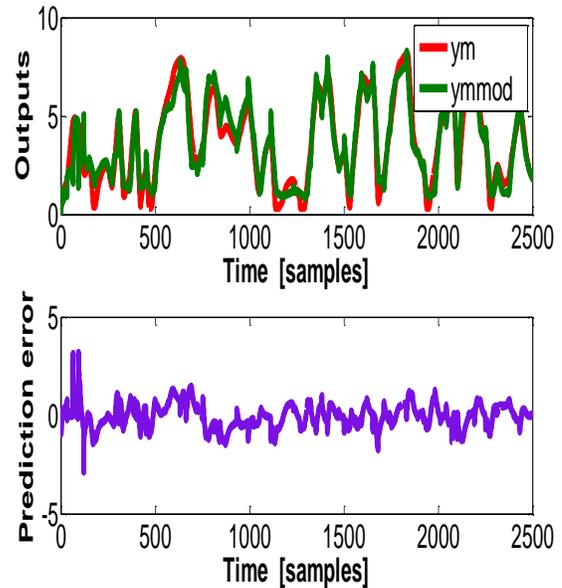


Figure 6 Model output with prediction error

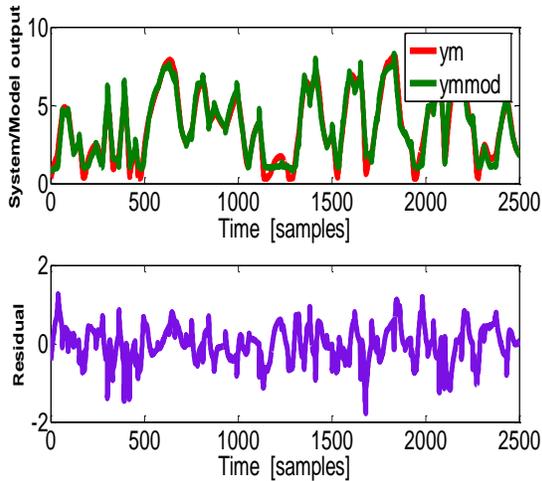


Figure 7 System/Model output with residuals

VI. CONCLUSION

The Wiener model are used in many domains for their simplicity and physical meaning, where the system steady-state behavior is determined completely by the static-nonlinearities, while the system dynamic behavior is determined by both the nonlinearities and the linear dynamic model components. Recursive Prediction Error Method allows the online identification of all linear model structure. RPEM is used to reduce the Mean Square Error (MSE). The Nonlinear modeling of the Cascaded two tank system using Wiener model is performed. Recursive Prediction Error Method using Wiener model is identified by considering the cascaded two tank data. In future, the recursive identification method will be extended to Hammerstein, Hammerstein-Wiener model and Uryson model.

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